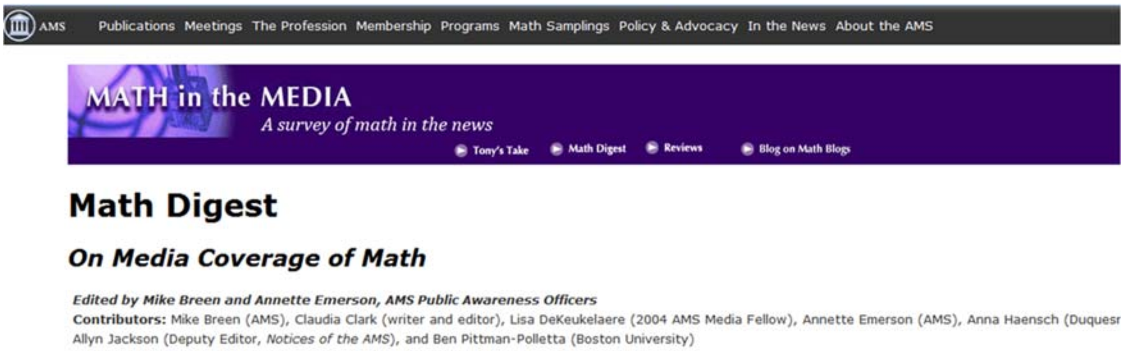
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MATH in the MEDIA

A survey of math in the news

Tony's Take Math Digest Reviews Blog on Math Blogs

Math Digest

On Media Coverage of Math

Edited by Mike Breen and Annette Emerson, AMS Public Awareness Officers

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A Famous Graph Makes an Appearance on a Very Small Stage, by Ben Pittman-Polletta

Imagine that you are an architect in a small, two-dimensional town--either planar or spherical--having only six buildings: three homes, and three utilities--a water plant, a gas plant, and an electric plant. You are trying to connect each home to each of the three utilities, but with a very strict aesthetic: you don't want to connect the homes serially - each home must have its own connection to each utility - and you don't want any of the connections to cross. The task you've set for yourself is the utilities problem, also known as the water, gas, and electricity problem. Go ahead and take a crack at it, I'll wait.

Welcome back. I hope you didn't spend a long time trying to draw those cables and pipes, because connecting the three houses to the three utilities without having a gas line cross a water pipe turns out to be impossible. Viewing the three houses and the three utilities as vertices of a graph, the connections between them form a complete bipartite graph, also known as the utility graph or K3,3. K3,3 is non-planar - that is, there is no embedding of this graph in a two-dimensional space of genus zero ("Why the Complete Bipartite Graph K3,3 is Not Planar", by Rod Hilton from his blog *Absolutely No Machete Juggling*, 29 October 2011), although it can be embedded in a torus. Not only is K3,3 nonplanar, it is in some sense one of only two nonplanar graphs. According to Kuratowski's theorem, a graph is nonplanar if and only if it contains a subgraph homeomorphic to either K3,3 or K5, the complete graph on 5 vertices.

Now imagine that you are a pregnant woman in the Congo during the '60s, looking for a medicinal tea to help you induce labor. Chances are, you'll reach for a medicinal tea that goes by the name kalata kalata, made from the plant *Oldenlandia affinis*. The active ingredient of kalata kalata is a peptide, named kalata B1. Kalata B1 is a ring of around 30 to 40 amino acids, interrupted at six places by the amino acid cysteine. The six cysteine residues are connected in pairs by three disulfide bonds. The six links between these cysteine residues--three disulfide bonds, and three chains of amino acids--make kalata B1 a protein incarnation of K3,3, with cysteine residues as vertices. In fact, kalata B1 is only one of a huge family of plant proteins known as cyclotides, all of which share the topology of K3,3. In these proteins, the linked cysteines are a constant, but the sections of amino acids between them are highly variable, containing different functional motifs. The cyclotides all share a remarkable rigidity and stability, thanks not only to their disulfide bonds but also to their peculiar topology, and a high level of resistance to digestion. They have potent insecticidal properties, and are being explored as a backbone for peptide drugs designed for oral administration (see "Cyclotide," Wikipedia.)

Finally, imagine you are polymer chemist Yasuyuki Tezuka. Polymers are macromolecules composed of many repeating subunits. Their behavior in aggregate--they may form materials that are tough, viscous, elastic, or combinations of all three--are dictated by their molecular properties. While many interesting things can be done with linear polymers--molecules made up of chains of subunits--you are interested in the unexplored frontier of cyclic polymers. You want to know how a plastic made of Hopf links or figure eights might behave. So, you develop a process allowing for the creation of molecules with simple but nontrivial topologies--such as a "theta" shape or an unfolded tetrahedron. Now you want to set your sights higher, to create a mathematically interesting as well as potentially useful cyclic polymer. What graph would you look to sculpt out of molecular bonds? As you've certainly guessed, Tezuka and his team set out to synthesize a tiny version of K3,3. They succeeded in part because K3,3 has an exceptionally compact 3D shape, when compared to other topological arrangements, allowing it to be isolated from these other molecules, and perhaps helping it to "achiev[e] exceptionally thermostable bioactivities" ("Constructing a

Macromolecular K3,3 Graph through Electrostatic Self-Assembly and Covalent Fixation with a Dendritic Polymer Precursor" by Suzuki, et al.). Tezuka credits his graduate student Takuya Suzuki, the paper's first author, with recognizing the utility of K3,3's compactness. "It's a very nice example of Japanese craftsmanship!" he says. But they aren't finished yet. "There are many other structures that are not easy to make at the nanoscale," he says. The "Konigsberg bridge-graph" appearing in their paper suggests what Tezuka's group might look to build next.

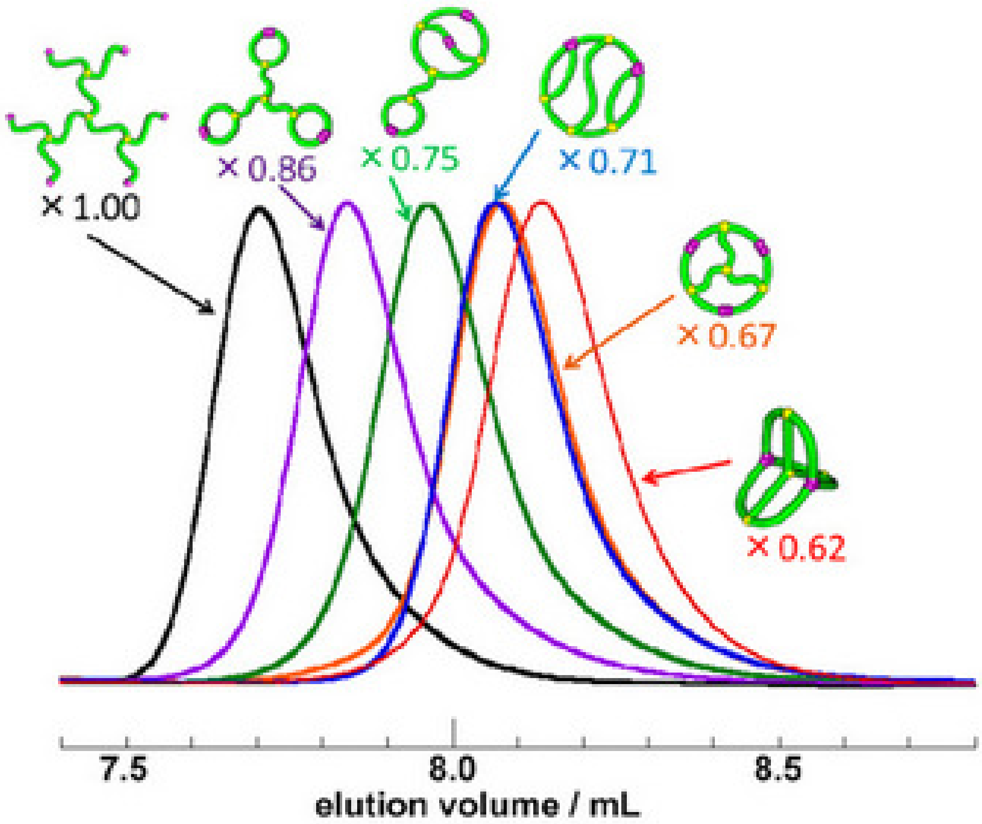

Image: The K3,3 graph, on the far right, has the smallest volume of all configurations shown, making it the fastest molecule in size-exclusion chromatography. Image courtesy of Dr. Yasuyuki Tezuka.

See "Materials scientists, mathematicians benefit from newly crafted polymers." *R&D Magazine*, 26 August 2014 (from *Tokyo Tech News*, 19 August 2014).

--- *Ben Pittman-Polletta* (posted 9/4/14)